BMJ EQUATIONS FOR DEEP CONVECTION IN WRF

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The equations shown in this section are the ones used in the BMJ scheme in WRF Version 3.3.1. They are based on Betts (1986) and Janjić (1994) that underlie the further developments in Fonseca et al. (2015) and Koh & Fonseca (2016).

In this cumulus scheme as explained in Betts (1986), the model first assesses whether there is Convective Available Potential Energy (CAPE) present and whether the cloud is sufficiently thick (i.e., $L_B - L_T > 2$ or $p_B - p_T > 10hPa$ where L_B and L_T are the cloud-base and cloud-top model levels and p_B and p_T the correspondent pressure levels; L_B is defined as the model level just above the Lifting Condensation Level (LCL) and has to be at least 25hPa above the surface whereas L_T is defined as the level at which CAPE is maximum (i.e., level of neutral buoyancy, LNB) for the air parcel with the maximum equivalent potential temperature θ_E in the depth $[PSFC, PSFC \times 0.6]$ where $PSFC$ is the surface pressure). If that is not the case there will be no convection and the scheme will abort. If either or both of those conditions are met, the cloud depth is compared to a minimum depth given by

$$D_{min} = 200hPa\left(\frac{PSFC}{1013hPa}\right) \quad (A1)$$

If the cloud depth is smaller than D_{min}, shallow convection is triggered; otherwise, deep convection is considered. In both shallow and deep convection (Betts, 1986), temperature and humidity fields are adjusted as follows

$$\begin{aligned} \Delta T_{BM} &= T_{REF} - T \\ \Delta q_{BM} &= q_{REF} - q \end{aligned} \quad (A2)$$

where ΔT_{BM} and Δq_{BM} are the Betts' adjustment of temperature T and specific humidity q in a model layer. Thus, the problem is reduced to defining the reference temperature and specific humidity reference profiles T_{ref} and q_{ref} for shallow and deep convection. In the BMJ scheme rainfall is only produced by deep convection which is the topic of this appendix.

RAINFALL

The BM scheme conserves enthalpy meaning that

$$\sum_{p_T}^{p_B}(c_P\Delta T_{BM} + L_{WV}\,\Delta q_{BM})\Delta p_L = 0 \qquad (A3)$$

where c_P is the specific heat at constant pressure for dry air assumed to be constant; L_{WV} is the latent heat of vaporisation for water vapour; Δp_L is the thickness of the model layer L in pressure coordinate. The total mass of water substance is conserved and hence in the original BM scheme (Betts, 1986) the precipitation is given by

$$\Delta P_{BM} = \frac{1}{g\rho_w}\sum \Delta q_{BM}\Delta p_L \qquad (A4)$$

where ρ_w is the density of liquid water; g is the acceleration of free fall.

In Janjić (1994), a parameter called cloud efficiency, E, is introduced and is defined as

$$E = c_1\frac{\overline{T}\Delta S}{c_P\,\sum \Delta T_{BM}\Delta p_L} \qquad (A5)$$

with

$$\overline{T} = \frac{\sum T_m \, \Delta p_L}{p_{bottom} - p_{top}}$$

$$\Delta S = \sum \left(\frac{c_P \, \Delta T_{BM} + L_{WV} \, \Delta q_{BM}}{T_m} \right) \Delta p_L$$

$$T_m = T + \frac{\Delta T_{BM}}{2}$$

where $\overline{T}$ is the weighted mean temperature of the cloudy air column; ΔS is the entropy change per unit area for the cloudy air column multiplied by g; T_m is the mean temperature over the time-step; c_1 is a non-dimensional constant estimated experimentally and set to 5. All summation symbols refer to summing over all cloudy layers $[L_B, L_T]$.

The denominator of (A5) is proportional to the single time-step rainfall from a model layer in the original BM scheme, (A4), and hence the cloud efficiency reduces when there is a propensity for heavy rain, partly correcting the tendency to over-predict intense rainfall in the original BM scheme.

In the default WRF-BMJ implementation, the precipitation, ΔP, and the adjustments in temperature and humidity, ΔT and Δq, over one cumulus time-step Δt are given by

$$\begin{cases} \Delta P = \Delta P_{BM} \, F(E) \;\; \Delta t/\tau \\ \Delta T = \Delta T_{BM} \, F(E) \, \Delta t/\tau \\ \Delta q = \Delta q_{BM} \, F(E) \, \Delta t/\tau \end{cases} \qquad (A6)$$

where $F(E)$ is a linear function of the cloud efficiency given by

$$F(E) = \left(1 - \frac{\Delta S_{min}}{\Delta S}\right) \left[F_1 + (F_2 - F_1)\left(\frac{E' - E_1}{E_2 - E_1}\right)\right] \qquad (A7)$$

with E' constrained to be in the range $[E_1, E_2]$:

$$E' = \begin{cases} E_1 & if \quad E \leq E_1 \\ E & if \; E_1 \leq E \leq E_2 \\ E_2 & if \quad E \geq E_2 \end{cases}$$

The constant $F_1 = 0.7$ is determined experimentally and $F_2 = 1$ for the chosen value of τ while $E_1 = 0.2$ is determined empirically in Janjić (1994) and $E_2 = 1$ for the chosen value of c_1. It is important to note that in Janjić (1994), $F(E)$ does not depend on the entropy change unlike the implementation we found in WRF version 3.3.1. In (A6) τ is the convective adjustment time-scale set to 40 min (Betts, 1986).

If the change in entropy is small (or even negative), i.e. $\Delta S < \Delta S_{min} = 10^{-4}\ JK^{-1}m^{-1}s^{-2}$, or very little (perhaps even negative) rainfall is obtained, i.e. $\sum \Delta T\ \Delta p_L \leq 10^{-7}\ Kkgm^{-1}s^{-2}$, shallow convection is triggered; otherwise, the BMJ scheme proceeds with deep convection. The reader is referred to Janjić (1994) for the documentation on shallow convection which we are not concerned with in this work.

REFERENCE PROFILES FOR DEEP CONVECTION

The first-guess potential temperature reference profile θ_{REF}^f for deep convection used in the BMJ scheme is assumed to have a vertical gradient that is a fixed fraction α of the vertical gradient of saturated equivalent potential temperature θ_{ES} following a moist virtual adiabat (i.e. isopleth of virtual equivalent potential temperature) from the cloud base up to the freezing level. Above the freezing level, θ_{REF}^f slowly approaches and reaches the environmental θ_{ES} at the cloud top. Thus, θ_{REF} given is prescribed by

$$\theta_{REF}^f(p_B) = \theta(p_0, T_0)$$

$$
\begin{cases} p_M \le p_L < p_B: & \theta_{REF}^f(p_L) = \theta_{REF}^f(p_{L-1}) + \alpha\,[\theta_{ES}(p_L) - \theta_{ES}(p_{L-1})] \\ p_T \le p_L < p_M: & \theta_{REF}^f(p_L) = \theta_{ES}(p_L) - \dfrac{p_L - p_T}{p_M - p_T}\{\theta_{ES}(p_M) - \theta_{REF}^f(p_M)\} \end{cases} \qquad \text{(A8)}
$$

where p_M denotes the pressure at the freezing model level, p_L denotes the pressure at any model level in the cloudy air column (such that L increases upwards from p_B to p_T) and p_0 and T_0 the pressure and temperature at the level from which the air parcel is lifted. θ_E is the equivalent potential temperature and e_B^* the saturated partial pressure of water vapour at the temperature T_B of the cloud base. In the first equation the constant α, according to Betts (1986), is equal to 0.85 but in the default WRF implementation it is set to 0.9, corresponding to a steeper $d\theta_{REF}/dp$ or a statically more stable profile. This choice of 0.9 for α was made when the scheme was tuned to the model over the North American region (Zaviša, pers. comm.).

The corresponding first-guess reference temperature profile is

$$
T_{REF}^f(p_L) = \theta_{REF}^f(p_L)\,\Pi(p_L) \qquad (A9)
$$

with

$$
\Pi(p_L) = \left(\frac{10^5\,\mathrm{Pa}}{p_L}\right)^{-R/c_p}
$$

where $\Pi(p_L)$ is the Exner's function (divided by c_P) for pressure p_L and R is the specific gas constant for dry air.

At pressure p_L equal or lower than 200hPa, the humidity field is not adjusted by the BMJ scheme. At pressure p_L larger than 200hPa in the convecting column, the first-guess reference specific humidity, $q_{REF}^f(p_L)$, is prescribed by the lifting condensation level, $p_L + \wp(p_L)$, of an air parcel with $\theta_{REF}(p_L)$ and $q_{REF}^f(p_L)$ at pressure p_L,

$$\begin{cases} q_{REF}(p_L) = q(p_L) & if\ p_L \le p_{200} \\ q^f_{REF}(p_L) = q^*\left(\theta^f_{REF}(p_L), p_L + \wp(p_L)\right) & if\ p_L > p_{200} \end{cases} \quad (A10)$$

where p_{200} is the pressure of a model level just smaller or equal to 200hPa. With the help of Tetens' formula (Tetens, 1930), the saturated specific humidity q^* is given by

$$q^*\left(\theta^f_{REF}(p_L), p_L + \wp(p_L)\right) = \left(\frac{379.90516\ \text{Pa}}{p_L + \wp(p_L)}\right) EXP\left\{17.2693882\left(\frac{\theta^f_{REF}(p_L) - \frac{273.16\ \text{K}}{\Pi\left(p_L + \wp(p_L)\right)}}{\theta^f_{REF}(p_L) - \frac{35.86\ \text{K}}{\Pi\left(p_L + \wp(p_L)\right)}}\right)\right\} \quad (A11)$$

The more negative $\wp(p_L)$ is, the drier the reference profile is at pressure level p_L. $\wp(p_L)$ is piecewise linearly interpolated between the values at the cloud bottom, $\wp_B$, freezing level, $\wp_M$, and cloud top, $\wp_T$, which are in turn parameterized as linear functions of cloud efficiency E as follows:

$$\wp_B = (-3875\ Pa)\left[F_S + (F_R - F_S)\left(\frac{E' - E_1}{E_2 - E_1}\right)\right] \quad (A12)$$

$$\wp_M = (-5875\ Pa)\left[F_S + (F_R - F_S)\left(\frac{E' - E_1}{E_2 - E_1}\right)\right] \quad (A13)$$

$$\wp_T = (-1875\ Pa)\left[F_S + (F_R - F_S)\left(\frac{E' - E_1}{E_2 - E_1}\right)\right] \quad (A14)$$

The constants in Pa above were determined by Janjic (1994) and are not varied in this work. In the WRF version 3.3.1 implementation, the parameter F_R is set to 1 while F_S is set to 0.85, an empirically determined value over continental USA (Zaviša, pers. comm.), while in the Janjić (1994) $F_S = 0.6$. Evidently, with a higher value of F_S, the formulation yields more negative $\wp(p_L)$ and a drier reference humidity profile for each cloud efficiency, $E < E_2$.

CONSERVATION OF ENTHALPY

To conserve enthalpy in the convecting column, the first-guess reference temperature and specific humidity profiles need to be corrected to yield the final reference profiles. The first-guess reference temperature is corrected by a constant T_ε to get:

$$T_{REF}(p_L) = T_{REF}^f(p_L) - T_\varepsilon \quad (A15)$$

For $p_L > p_{200}$,

$$q_{REF}(p_L) = q^*\left(\theta_{REF}(p_L), p_L + \wp(p_L)\right) \quad (A16)$$

The first-guess reference specific humidity profile is given by (A10). Taking the difference between (A16) and (A10) and using (A9), we define

$$q_\varepsilon = q_{REF}^f(p_L) - q_{REF}(p_L) = \left(\frac{\partial q^*}{\partial \theta}\right)_p \left(\theta_{REF}^f(p_L) - \theta_{REF}(p_L)\right)$$

$$\Leftrightarrow q_\varepsilon = \left(\frac{\partial q^*}{\partial T}\right)_p \left(T_{REF}^f(p_L) - T_{REF}(p_L)\right) = \left(\frac{\partial q^*}{\partial T}\right)_p T_\varepsilon = \gamma\, T_\varepsilon \quad (A17)$$

where

$$\gamma = \left(\frac{\partial q^*}{\partial T}\right)_p \cong q_{REF}^f(p_L) \frac{4098.03\ \mathrm{K}}{\left(\theta_{REF}^f(p_L)\Pi\left(p_L + \wp(p_L)\right) - 35.86\ \mathrm{K}\right)^2}$$

By combining equations (A15) and (A17) with (A2), we can write

$$\Delta T_{BMJ} = \Delta T^f - T_\varepsilon$$

$$\Delta q_{BMJ} = \Delta q^f - q_\varepsilon$$

where

$$\Delta T^{f} = T^{f}_{REF}(p_L) - T(p_L)$$

$$\Delta q^{f} = q^{f}_{REF}(p_L) - q(p_L)$$

Conservation of enthalpy, (A3), requires that

$$\sum_{p_T}^{p_B} \left(c_P \Delta T_{BMJ} + L_{WV}\, \Delta q_{BMJ} \right) \Delta p_L = 0$$

$$\Leftrightarrow \sum_{p_T}^{p_B} (c_P\, \Delta T^{f} + L_{WV}\;\; \Delta q^{f}) \Delta p_L = \sum_{p_T}^{p_B} (c_P T_{\varepsilon} + L_{WV}\, q_{\varepsilon}) \Delta p_L$$

$$\Leftrightarrow\; T_{\varepsilon} = \frac{\sum_{p_T}^{p_B} (c_P\, \Delta T^{f} + L_{WV}\;\; \Delta q^{f}) \Delta p_L}{\sum_{p_T}^{p_B} (c_P + L_{WV}\, \gamma) \Delta p_L}$$

The cloudy column above 200hPa (if the cloud top is high enough in the first place) is treated separately from the rest of the cloudy column as the humidity field is not adjusted by the BMJ scheme when the pressure p_L is equal or lower than 200hPa (i.e. $\Delta q_{BMJ} = 0$). Hence, the correction to the first-guess reference temperature, T_{ε}, is given by

$$T_{\varepsilon} = \frac{\sum_{p_B}^{p_T} (c_P\, \Delta T^{f} + L_{WV}\, \Delta q^{f}) \Delta p_L}{\sum_{p_B}^{p_{200}} (c_P + L_{WV}\, \gamma\,) \Delta p_L + \sum_{p_{200}}^{p_T} c_P \Delta p_L} \qquad (A18)$$

With the reference temperature and specific humidity profiles defined through equations (A8), (A9), (A10), (A11), (A15) and (A18), the convective adjustment in temperature and specific humidity over one time-step can be computed by equations (A2) and (A6).

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